

B.Sc. Semester - 1 (CBCS) Examination
Jan/Feb.-2022 (NEW COURSE)
MATHEMATICS (CORE)

Time: 1:30 Hours

Marks: 42

Instructions:

1. Figures to the right indicate marks.
2. There are five questions in the question paper.
3. Answer any three of the following questions.

Q.1 (A) Answer any two of the following questions.**[10]**

- (1) Give an example to show that conditions of Roll's theorem are sufficient but not necessary.
- (2) Let $b > a > 0$. Show that there exists $c \in (a, b)$ such that $b^b - a^a = c^c [b \log b - a \log a]$.
- (3) Find Taylor's series expansion of the function $f(x) = \log x$ near $a = 1$.

Q.1 (B) Answer any one of the following questions.**[04]**

- (1) State and prove Lagrange's mean value theorem.
- (2) Find Maclaurin's series expansion of the function $f(x) = \cos x$, ($x \in R$).

Q.2 (A) Answer any two of the following questions.**[10]**

- (1) If $\lim_{x \rightarrow 0} \frac{(1+a \cos x) - b(1-2x^2)}{x^2} = 1$, then find values of a and b .
- (2) Evaluate: $\lim_{x \rightarrow 0} \frac{1 + \sin x - \cos x + \log(1-x)}{x \tan^2 x}$.
- (3) Solve $(2x + 3y - 4) dx + (3x + y + 5) dy = 0$.

Q.2 (B) Answer any one of the following questions.**[04]**

- (1) Evaluate: $\lim_{x \rightarrow 0} \left(\frac{a^x + b^x + c^x}{3} \right)^{\frac{1}{x}}$.
- (2) Solve $x(x - y) dy + y^2 dx = 0$.

Q.3 (A) Answer any two of the following questions.**[10]**

- (1) Discuss the procedure of solving differential equation which is solvable for x by considering suitable example.
- (2) Solve $\frac{d^4 y}{dx^4} + 5 \frac{d^2 y}{dx^2} - 36y = 0$.
- (3) Solve: $p^2 - 7p + 12 = 0$

Q.3 (B) Answer any one of the following questions.**[04]**

- (1) Solve: $y = p \sin p + \cos p$.
- (2) Write general form of n^{th} order linear differential equation and explain complementary function and auxiliary equation.

Q.4 (A) Answer any two of the following questions.**[10]**

- (1) Find general solution of the differential equation $(D^2 - 6D + 9)y = e^{3x}$.
- (2) Find general solution of the differential equation $(D^2 + 2D + 1)y = 2x + x^2$.
- (3) Evaluate $\frac{1}{D^2 - 2D + 1} x^2 e^{3x}$.

Q.4 (B) Answer any one of the following questions.**[04]**

- (1) In usual notations prove that $\frac{1}{f(D)} e^{ax} = \frac{1}{f(a)} e^{ax}$ if $f(a) \neq 0$.
- (2) Evaluate $\frac{1}{D^3 + 4D} \sin 2x$.

Q.5 (A) Answer any two of the following questions.**[10]**

- (1) Find P. I. of $(x^2 D^2 - 2)y = x^2 + \frac{1}{x}$.
- (2) Solve: $(x^2 D^2 + 4xD + 2)y = e^x$.
- (3) Solve: $x^2 \frac{d^2 y}{dx^2} + y = 3x^2$.

Q.5 (B) Answer any one of the following questions.**[04]**

- (1) In usual notations prove that $\frac{1}{f(\theta)} x^m = \frac{1}{f(m)} x^m$ if $f(m) \neq 0$.
- (2) Solve : $(x^2 D^2 + 3xD + 1)y = 0$.
